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CHARACTERIZATION OF DIFFERENT REGIMES IN NONLINEAR LIQUID CRYSTAL MODELS

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The range of validity of two models for nonlocal nonlinear optics in Nematic Liquid Crystals (NLC) is studied. Particularly the influence of the optical power and the initial position of the beam over its trajectory is studied when launching the beam with an offset in a planar cell. The main difference between both models is the dependence of the orientational angle with the optical field, either linear or nonlinear. The results demonstrate the critical role of the nonlinearity in the propagation of nematicons in NLC planar cells.

Keywords: Liquid Crystals; Nematics; Nonlinear optics.

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1. Introduction

Nematic Liquid Crystal (NLC) devices are being widely studied in the field of nonlinear optics due to its large nonlinear response.[1,2] It allows to generate nonlinear solutions with no change of shape, the so called nematicons at very low optical powers.[3] Numerous applications are being studied manipulating these nematicons: micro-devices for steering solitons in NLCs using curved surfaces[4] or external light sources,[5] all-optical switching and logic gating with spatial solitons in liquid crystals[6,7] and signal processing by opto-optical interactions between self-localized and free propagating beams in liquid crystals,[8] for example. Besides, the nonlocality exhibited by NLC cells has been shown as an efficient mechanism for stabilizing optical complex solutions which cannot exist in local nonlinear homogeneous media, as it has been demonstrated numerically in the cases of vortex and multivortex solitons,[9–11] dipole solitons[12] or soliton clusters[13] as well as in experiments like those described in Refs. 14 and 15.

The transverse oscillation of a nematicon induced by the force exerted by the boundary in highly nonlocal nonlinear media when there exists an asymmetry on the launching position of the beam is a well-known phenomenon as it has been pointed in Ref. 16 and experimentally in Ref. 17, as well as for thermal media in Ref. 18. The effect of the boundary force can be seen when the initial position of the light beam is not centered in the cell since it yields an offset in the peak position of the tilt angle which does not coincide with the position of the optical peak. We pretend here to show how the orientational nonlinearity affects this transverse oscillation, since linearized models are usually employed to explain certain properties of the nematicon trajectory such as its period or its amplitude.

Orientational nonlinearity arises, in the more complete nonlinear model we study in this work, by the effective coupling among the three differential equations involved and not from a nonlinear term appearing in the beam propagation method as other commonly used models do.[19] Here we are not restricted to low tilt angle values as it occurs with linearized models.[17]

The contents of the paper are as follows: Sec. 2 shows the two models which are being compared in this work explaining their derivation while in Sec. 3 a numerical comparison studying the transverse oscillation of nematicons is presented. Finally our main conclusions are summarized in Sec. 4.

2. Physical Models

Let us consider a NLC device consisting of two glass plates with commercial NLC E7 in between and two indium tin oxide (ITO) transparent electrodes next to the glass plates which permits us to apply external voltages to the liquid crystal. Figure 1 illustrates the geometrical configuration of the NLC device, where the z co-ordinate is the propagation direction, while x and y co-ordinates represent transverse spatial dimensions.[17] Planar alignment imposes director orientation nearly parallel to the glass plates, i.e., tilt angles smaller than two degrees (2). Let us take the optical field

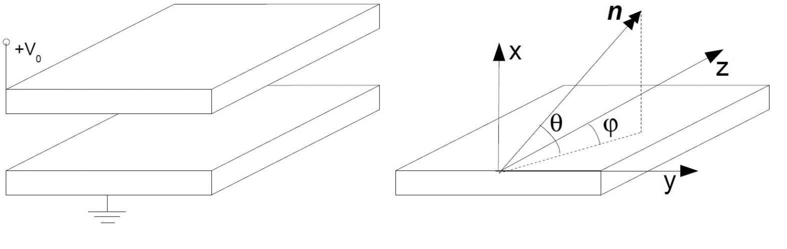


Fig. 1. NLC device. Cell with planar alignment (left). On the right we see the parameters that define the director orientation: the tilt and twist angles.[17]

to be linearly polarized as an extraordinary wave in direction x. The optical axis (or molecular director) of the NLC is parallel to z and no twist angle is allowed in the model, since the symmetry in the transverse co-ordinate y of the electrodes does not allow the appearance of y components in the electric field, which are responsible for twist deformations. We will focus in lateral propagation of light inside the device, this is, the wavevector nearly parallel to propagation direction, z co-ordinate. No big deviation angles are allowed since we work with paraxial approximation. Model B (Mod. B) was introduced in the first paper on nematicons.[20] Then, the setup has been being widely studied in the highly nonlocal nonlinear regime.[21–23]

The two models of NLC devices to be studied in this paper are the following. Model A (Mod. A) is the linear model employed in other studies[24,25] which assumes two main hypothesis. First, the orientational equation is linearized, so the model is suitable only for low tilt angle deviations, around a tilt angle of $\pi/4$, which maximizes the nonlinear effects. And second, the external electric field distribution will not be introduced in the model so it will not be possible to study the influence of external fields. The resulting governing equations are given by:

$$K\frac{\partial^2\theta}{\partial x^2} + 2\epsilon|E_x^o|^2 = 0, \tag{1}$$

$$j\frac{\partial E_x^o}{\partial z} = \frac{1}{2}\frac{\partial^2 E_x^o}{\partial x^2} + 2\theta E_x^o, \tag{2}$$

where θ is the orientational angle, E_x^o is the amplitude of the optical field linearly polarized through x direction, K is the distortion energy constant, $j = \sqrt{-1}$ and ϵ is the dielectric constant of the medium.

Equation (1) is solved analytically in Ref. 16 for studying the transverse oscillation of a nematicon under the effect of boundary forces when the optical beam is launched with an offset in the transverse co-ordinate. Ehrenfest theorem is employed there in order to study the boundary force over the nematicon as well as its trajectory and period dependence over the optical power and offset arriving to conclusions on the approximate behavior of such phenomenon.

Model B was introduced in the first paper on nematicons[20] and is a nonlinear model mainly that has been also studied, among others, by Beeckman *et al.*[3,22,26] The two hypothesis used in Mod. A are relaxed, so the full nonlinear character of the orientational equation is used and the external electric fields are included. Here, just scalar and paraxial approximations are done on the beam propagation method presented above. The system of equations of this nonlinear models is

$$K\frac{\partial^2\theta}{\partial x^2} + \frac{1}{2}\epsilon_0\sin(2\theta)(\Delta\epsilon^s|E_x^s|^2 + \Delta\epsilon^o|E_x^o|^2) = 0, \tag{3}$$

$$\int_0^d E_x^s dx = \int_0^d \frac{D_x^s}{\epsilon_{xx}} dx = -V_0, \tag{4}$$

$$2jk_0n_0\frac{\partial E_x^o}{\partial z} = \frac{\partial^2 E_x^o}{\partial x^2} + k_0^2(\epsilon - n_0^2)E_x^o, \tag{5}$$

where E_x^s is the static electric field (just x component), D_x^s is the corresponding displacement field, V_0 is the potential applied to the electrode, $k_0 = 2\pi/\lambda_0$ is the wavevector of the light beam, n_0 is the refractive index, ϵ_0 is the vacuum dielectric constant, $\Delta\epsilon^s = \epsilon_\| - \epsilon_\perp$ is the birefringence at external field frequencies equal to the difference between the relative (a dimensional) permittivities for parallel and perpendicular directions respect to z, $\Delta\epsilon^o = n_\|^2 - n_\perp^2$ is the optical birefringence equal to the difference between the refraction indices for parallel and perpendicular directions, $\epsilon(\theta)$ is the nonlinear optical permittivity and $\epsilon_{xx}(\theta)$ is the nonlinear electrical permittivity. Let us take $\epsilon = \epsilon_\perp^o + \Delta\epsilon^o \sin^2\theta$, and $\epsilon_{xx} = \epsilon_\perp^s + \Delta\epsilon^s \sin^2\theta$ for the nonlinear permittivities.

In this paper the properties of NLC $E7$ are used: $K = 12 \times 10^{-12}\,\mathrm{N}$, $\epsilon_\| = 19.6$, $\epsilon_\perp = 5.1$, $n_\| = 1.6954$, and $n_\perp = 1.5038$. Hence, $\Delta\epsilon^s = 19.6 - 5.1 = 15.4$, and $\Delta\epsilon^o = n_\|^2 - n_\perp^2 = 1.6954^2 - 1.5038^2 = 0.613$. We also have used $n_0 = 1$, $\lambda_0 = 1.064\,\mu\mathrm{m}$ and $\epsilon_0 = 8.8541878176 \times 10^{-12}\,\mathrm{CV^{-1}m^{-1}}$.

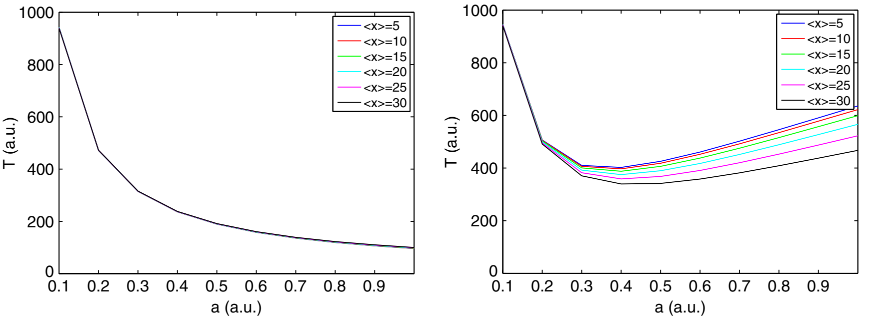
These models are solved numerically in an iterative way so that an auto-consistent solution is determined.[17,26] The procedure is as follows: first, find an auto-consistent solution for static Eqs. (3) and (4). Second, use such solution to calculate the next propagation step in Eq. (5). And third, since this optical field influences again the whole static problem, repeat the first two steps until convergence. Note that Eq. (4) has an analytical solution. However (3) and (5) are solved numerically by a standard finite difference time domain (FDTD) method.[27] Since Eq. (3) is nonlinear an iterative scheme is required, solving a linear system of equations obtained by the FDTD method in every step. Equation (5) is linear so no iteration is required.

3. Comparison Between the Models

Our goal is to check to which extent the linear approximation is valid when studying the transverse oscillation mentioned above. In order to compare the results of each model, we take Mod. B to the range of parameters at which it is equivalent to Model A, this is taking $k_0 = 1$, $\epsilon_\perp = 1$, $\Delta\epsilon = 4$, $K = \nu\Delta\epsilon\epsilon_0\theta_{\max}/2$, $\theta \ll 1$, and $\theta_{\max} = 1/2$.

The amplitude of this transverse oscillation depends on the initial position of the beam with its period depending on the optical power and the offset of its initial position.

The dependence on the period of such oscillation on the optical amplitude is shown in Fig. 2 for both models. We can see there are different behaviors between the models. Figure 2(a) shows that the period of the oscillations for Model A evolves monotonically decreasing with the square root of the power density (since the force exerted by the boundary is linear with the power density). For Mod. B, Fig. 2(b) shows a nonmonotonic behavior, with a minimum period close to an amplitude of 0.4 for every value of the offset, as expected since the linear relationship between intensity and index perturbation is no longer valid. Figure 2 also illustrates that

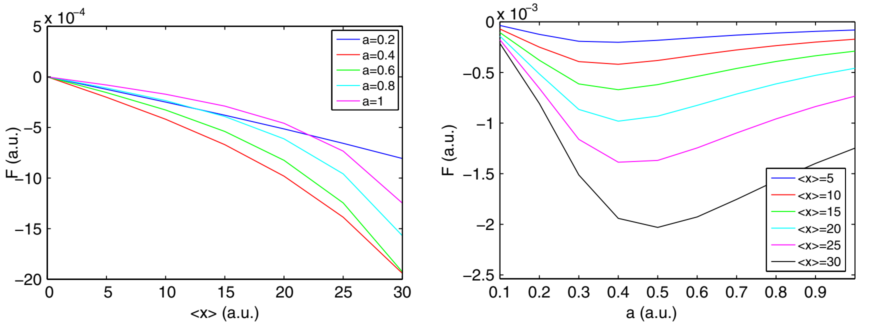


(a) (b)

Fig. 2. Period of the transverse oscillation T versus optical amplitude a for (a) Model A and (b) Model B.

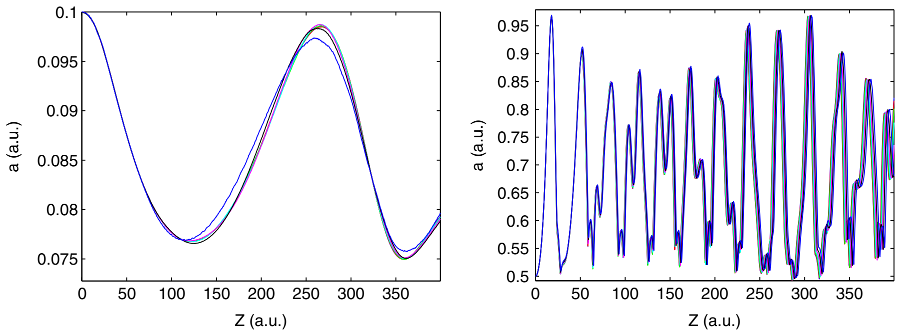
the initial position does not influence the period of the transverse oscillation for Model A but they do for Mod. B, as stated in Ref. 16 and experimentally in Ref. 17 as well as in the case of thermal media.[18] Model A shows very little dependence of the period over the offset. We can see the nonlinearity of Mod. B induces this minimum, which can be explained in terms of the nonlinear force exerted by the boundary. This force has been calculated numerically through the second derivative of the trajectory of the center of mass. It is known that the force grows when approaching the boundary.[17] Figure 3 shows the dependence on the initial position of the force and its dependence with the optical amplitude. There it can be seen the nonlinearity of the relationship among the force and the optical amplitude, which makes it appear as a minimum responsible of the minimum appearing in the period dependence on the optical amplitude.

The evolution of the amplitude peak of the optical field is shown in Figs. 4 and 5. We see that results are qualitatively (even quantitatively) equal for low powers



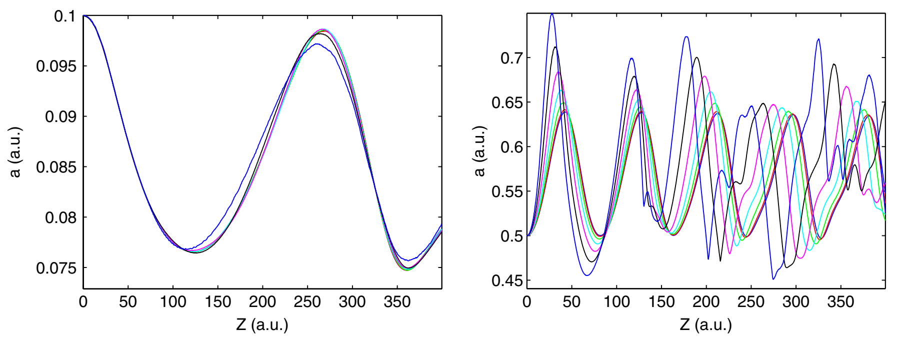
(a) (b)

Fig. 3. (a) Force F exerted by the boundary versus initial position $\langle x \rangle$ for different optical amplitudes, a. (b) Force F versus optical amplitude a for different offset values $\langle x \rangle$.



(a) (b)

Fig. 4. Evolution of the optical peak amplitude a for (a) $a = 0.1$ and (b) $a = 0.5$ using Model A. Each color represents different values for the offset $\langle x \rangle$.



(a) (b)

Fig. 5. Evolution of the optical peak amplitude a for (a) $a = 0.1$ and (b) $a = 0.5$ using Mod. B. Each color represents different values for the offset $\langle x \rangle$.

where the linear approximation is valid ($a = 0.1$). However differences arise as power increases. We observe three main differences. First, the oscillation frequency of the peak amplitude increases for Mod. A as it does the optical amplitude but does not for Mod. B. This is closely related to the artificial higher values of the director orientation reached in Mod. A. Higher values of the midtilt yields higher values for the effective index. Model A establishes a linear relationship among the potential, or effective index profile, and the midtilt, the value of the tilt at its maximum, as it can be seen in Eq. (2). There it can be seen that the potential term is 2θ. However Mod. B has a potential term which is proportional to the dielectric permittivity $\epsilon = \epsilon_\perp + \Delta\epsilon \sin^2\theta$ (Ref. 1) which yields in a bounded saturable value for the midtilt that makes it reach more moderate realistic values than Mod. A. Moreover the greater the values of the effective index the bigger will be the self-focusing effect which explains the high value for the frequency of the optical amplitude oscillation.

Second, Mod. A almost overlap the amplitudes for different offset values, what can be best appreciated in Fig. 4, however, Mod. B shows a different behavior of the peak amplitude when changing the offset (see Fig. 5) since its frequency is much lower and behaviors for different offset values are not so equal as it occurs in Mod. A.

And third, the difference between its maximum value and its minimum one is much higher in Mod. A than in Mod. B as it appears a higher self-focusing effect in Mod. A due to a non-saturable effect of the midtilt. Such effect is an artifact of the linear approximation to the orientational nonlinearity, which is saturable as the midtilt is bounded by $\pi/2$.

We must also mention that the presence of transverse oscillations in solitons in nematics was first reported and studied by Peccianti *et al.*[28] and their breathing in Ref. 29. Our results show a good agreement in the weak nonlinear regime. A full comparison in the high nonlinear regime is beyond the scope of the present work and will be addressed in further research.

4. Conclusions

In conclusion, we have investigated the limits of standard liquid crystal models when applied to study a transverse oscillation phenomenon induced by the boundary. Big differences appear when we work in the nonlinear regime where the simplified models are no longer valid. Numerical simulations are in good agreement with related results already published at the time they widen our knowledge of such transverse oscillation.

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