Journal of Modern Optics
Vol. 55, No. 17, 10 October 2008, 2819–2829

Laser tweezers for atomic solitons

A.V. Carpentier[a*], J. Belmonte-Beitia[b], H. Michinel[a] and M.I. Rodas-Verde[a]

[a]*Área de Óptica, Facultade de Ciencias, Universidade de Vigo, Ourense, Spain;* [b]*Departamento de Matemáticas, E.T.S.I. Industriales, and Instituto de Matemática Aplicada a la Ciencia y la Ingeniería (IMACI), Universidad de Castilla-La Mancha, Ciudad Real, Spain*

(*Received 25 February 2008; final version received 15 May 2008*)

We describe controllable and precise laser tweezers for Bose–Einstein condensates of ultracold atomic gases. In our configuration, a laser beam is used to locally modify the sign of the scattering length in the vicinity of a trapped BEC. The induced attractive interactions between atoms allow us to extract and transport a controllable number of atoms. We analyze, through numerical simulations, the number of emitted atoms as a function of the width and intensity of the outcoupling beam. We also study different configurations of our system, as the use of moving beams. The main advantage of using the control laser beam to modify the nonlinear interactions in comparison to the usual way of inducing optical forces, i.e. through linear trapping potentials, is to improve the controllability of the outcoupled solitary wave-packet, which opens new possibilities for engineering macroscopic quantum states.

Keywords: Bose–Einstein condensate; solitons; laser tweezers

1. Introduction

The achievement of Bose–Einstein condensation (BEC) in dilute gases of alkali atoms [1,2] has driven the research on the design of new tools for the manipulation and coherent control of atomic ensembles. In the last years, an intensive study of different mechanisms for this purpose has been carried out both theoretically and experimentally. Among the most important contributions we must cite the realization of atom mirrors [3], guides [4,5], the design of atomic accelerators both in linear and circular geometries [6–8]. Atom lasers have also been developed, first based on the use of short radio-frequency pulses as an outcoupling mechanism, flipping the spins of some of the atoms to release them from the trap [9]. Later, other coherent atomic sources were built leading to pulsed, semicontinuous or single-atom lasers [10–15].

The control of coherent atomic beams is a challenging problem in physics due to its potential applications in multiple fields like atom interferometry [16], superposition of quantum states [17], atom clocks [18], or quantum information [19], among others. Many of these devices take advantage of nonlinear interactions between atoms, which are ruled by the value of the scattering length a. The adequate tuning of this parameter has been possible with the use of Feshbach resonances [20] and has yielded impressive

*Corresponding author. Email: avcarpentier@uvigo.es

ISSN 0950–0340 print/ISSN 1362–3044 online

DOI: 10.1080/09500340802209763
http://www.informaworld.com

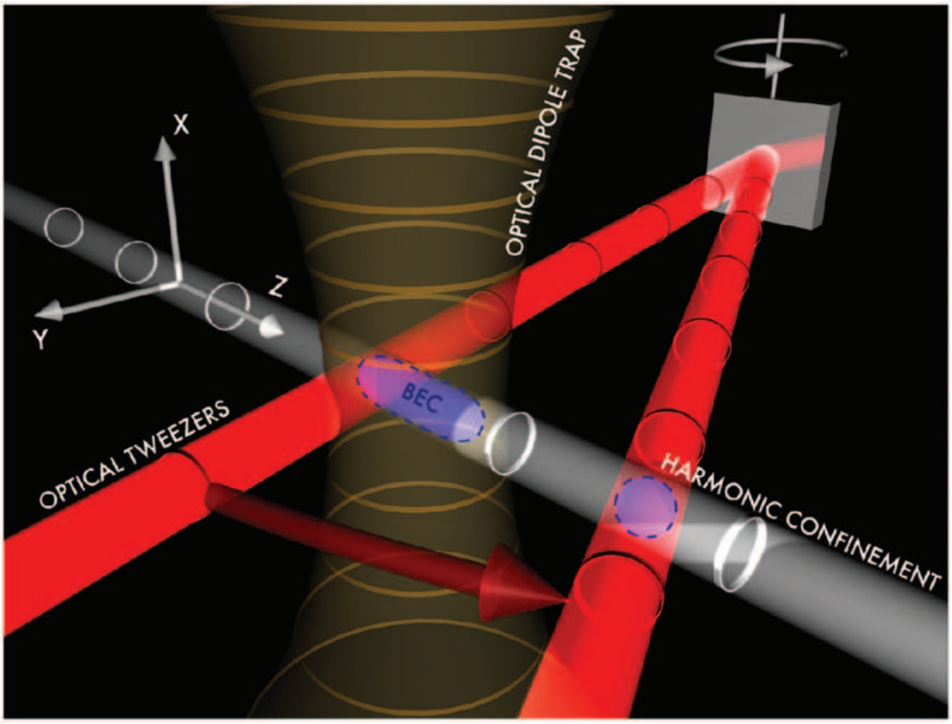

Figure 1. Sketch of the system we will study in this paper for the case of optical tweezers which are used to extract a given number of atoms from a BEC reservoir. A rotating mirror is used to move a laser beam over a BEC which is transversely trapped by magnetic confinement, and optically along the direction of motion of the laser. (The color version of this figure is included in the online version of the journal.)

effects like the macroscopic collapse of matter waves [21] or the creation of atomic solitons [22–24]. The recent realization of optical control of Feshbach resonances [25] has paved the way for the experimental demonstration of many theoretical proposals on nonlinear waves in Bose–Einstein condensates with spatially inhomogeneous interactions [26–29] including the dynamics of solitons when the modulation of the nonlinearity is a random [30,31], linear [32], periodic [33–35], localized function [36] or step-like function [37].

In this paper, we propose the use of spatially-dependent scattering length as a tool for designing precise atom tweezers which are able to extract a portion of atoms from a BEC (see Figure 1). Other methods have been proposed for this purpose, as in [38] and [39] in combination with spatial light modulators for splitting the atomic cloud. Our device is inspired by a coherent atomic source based on the spatial modification of the scattering length [40,41], which produces a highly regular and controllable number of atomic pulses by modulating a along the trapping axis of a BEC. As we will show in this paper, in comparison with linear traps which do not alter the value of a, spatially-tailored nonlinear interactions yield robust control of the number of atoms that can be extracted from a BEC reservoir, providing new ways to the creation of macroscopic superposition of quantum states from arrays of Bose–Einstein condensates [42].

The structure of this paper is as follows: in Section 2, we introduce the configuration of the system and the mean field theory which is used in this work. In Section 3 we study by means of numerical simulations the mechanism of emission by employing static laser tweezers. We also analyze the number of atoms emitted as a function of the main parameters of the system. Finally, in Section 4, we analyze the use of moving tweezers which trap atoms by crossing the condensate, and compare our results with the linear case in which there is no spatial variation of the scattering length.

2. System studied and model equations

We will assume that a BEC is strongly trapped in the transverse directions (x, y) and weakly confined along the longitudinal one (z) leading to a *cigar shaped* configuration. We will consider the effect of a spatial variation of the scattering length along z which can be switched from positive to negative values by the optical control of Feshbach resonances by means of a laser beam. The region of negative scattering length can be varied in size and displaced along z by simply focusing or moving the laser beam. The choice of an optical control [25] instead of a magnetic one [20] allows for a faster and easier manipulation of the spatial variations of the scattering length.

The mean field description of the dynamics of the BEC is provided by a Gross–Pitaevskii (GP) equation of the form:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi + V(r)\Psi + U|\Psi|^2\Psi, \tag{1}$$

where Ψ is the order parameter, normalized to the number N of atoms in the cloud: $N = \int |\Psi|^2 \, d^3r$. $U = 4\pi\hbar^2 a/m$ characterizes the two-body interactions determined by the value of the scattering length a. The cloud of N equal bosons of mass m is tightly trapped in (x, y) by a harmonic potential $V_\perp$ of frequency $\nu_\perp$ and weakly confined along z by the effect of an optical dipole trap V_z that can be produced by a laser beam of a given width along z [43,44]. The mathematical expression for the potential is thus:

$$\begin{aligned} V(r) &= V_\perp + V_z \\ &= \frac{m\nu_\perp^2}{2}\left(x^2 + y^2\right) + V_\mathrm{d}\left[1 - \exp\left(-\frac{z^2}{L^2}\right)\right], \end{aligned} \tag{2}$$

where V_d is the depth of the (shallow) optical dipole potential and L its characteristic width along z. To fix ideas, we will present specific numbers in this paper corresponding to ^{7}Li, using the experimental parameters of [23], where $V_\mathrm{d} \approx \hbar\nu_\perp$, with $\nu_\perp = 1$ kHz being the frequency of the trap in the transverse plane (x, y), which yields to a transverse radius $r_\perp \approx 3\,\mu$m. The other numerical values used in our simulations are $L = 4r_\perp$, $N = 3 \times 10^5$, $w_\mathrm{c} = 5.4r_\perp$ (which is the longitudinal size of the BEC cloud) and $a = -1.4$ nm. We must stress that our basic ideas should hold for different atomic species like ^{85}Rb and ^{133}Cs with an adequate change of the parameters used.

3. Atom extraction with static tweezers and control of the atomic wavepacket

The problem we will face is the controllable extraction of atoms from a BEC reservoir. We will consider a system configuration in which a trapped cigar-type condensate is partially overlapped by a laser beam in a similar configuration to the one described in [40]. Under adequate conditions, the laser changes the local value of the scattering length in part of the cloud. If a is locally switched to large enough negative values, a burst of atomic solitons can be emitted from the condensate. If the distance between the laser beam and the center of the cloud is kept constant, an emitted soliton may rebound inside the laser beam and thus remain trapped out of the reservoir [41]. Once the atoms are extracted, the laser can be moved away from the condensate. This also allows us to control the position of the soliton in the z direction. This idea of extracting atoms is radically different than using a usual dipole trap to extract atoms without switching the scattering length to negative values. In the latter case, the atoms will perform Josephson oscillations between the reservoir and the tweezers and it is only possible to extract a significant portion only at times exactly matching the maxima of the periodic motion. Thus, the role of nonlinear interactions is essential in this static configuration to guarantee that once extracted the atoms will not go back to the reservoir.

Our results are based on numerical simulations of Equation (1). All results to be presented in what follows have been obtained using second order in time split-step pseudospectral solvers, with the spatial derivatives being evaluated by Fourier methods [45]. In Figures 2(a) and (b) we show some numerical simulations showing how our nonlinear tweezers work. In both pictures, we have plotted pseudocolor images of the cloud density. The horizontal axis is z and vertical axis is time. The extraction of atoms is made with a Gaussian-profile laser beam. If the scattering length is switched to large-enough negative values, part of the cloud is extracted. Once the atoms have left the reservoir, the laser beam is moved away from the condensate dragging part of the atoms. In Figure 2(a) the laser beam extracts one soliton and controls its position along z. In Figure 2(b) two solitons are extracted with two different laser beams and their paths joined at a given point. Time in the vertical axis goes from $t=0$ to $t=500/\nu_\perp$ in Figure 2(a) and from $t=0$ to $t=1800/\nu_\perp$ in Figure 2(b), where $\nu_\perp=1\,\text{kHz}$ is the radial trapping frequency. The horizontal axis spans 60 times the width L of the optical dipole trap that confines the condensate in the z direction. The figures on the right represent the atom density showing the profiles of the reservoir and the emitted solitons (in black continuous lines), and of the laser beams (in dotted black lines and shaded) for three different times indicated with dashed lines in Figures 2(a) and (b). As it can be appreciated our control method allows a robust control of the extracted atoms.

Let us now consider a slightly different configuration in order to show the robustness of the method. We now address the case of a laser beam which is more intense and narrow than in the previous simulations. In this case a high nonlinear interaction stripe is generated and some atoms will be attracted to this thin region of negative scattering length and will be trapped. As in the previous case, the position of the extracted atoms along z can be controlled by moving the laser. In Figures 3(a) and (b) we show a similar representation as in Figures 2(a) and (b). In this case, a much narrower beam has been used in order to suppress internal rebounds of the extracted atomic beam. In Figure 3(b) two matter waves are emitted employing two different laser beams. By displacing the lasers, it is possible to control the relative position of the extracted solitons. Vertical axis corresponds to times in the range from $t=0$ to $t=5000/\nu_\perp$ for both figures. The plots at the right display the profiles of the condensate and emitted solitons (in black continuous lines) and of the

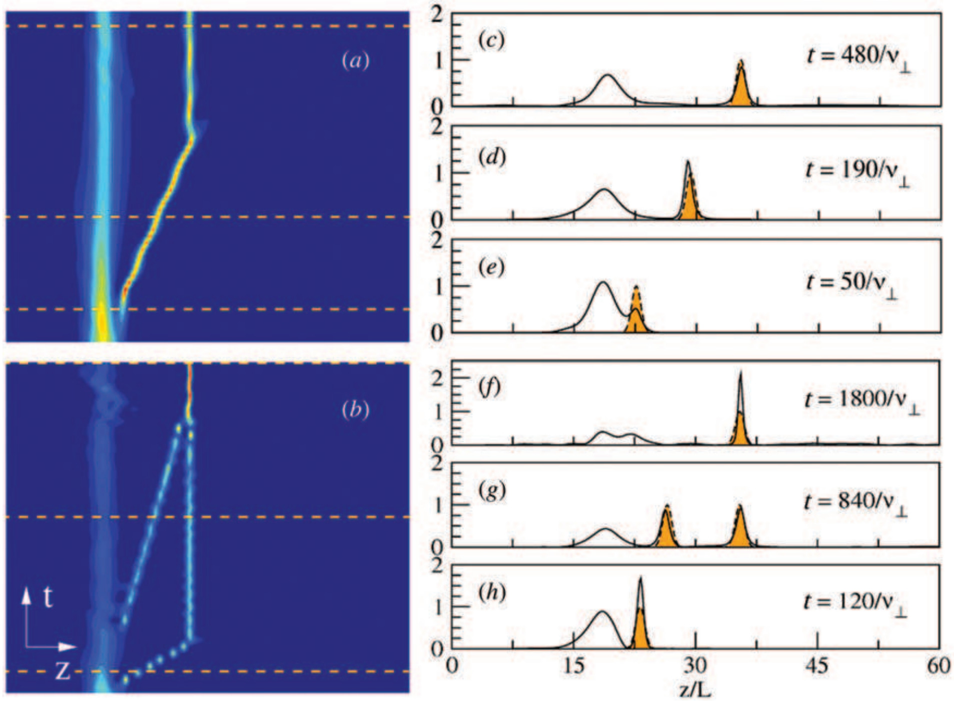

Figure 2. Controlled emission of atomic solitons from a BEC reservoir. The extraction was made by employing a Gaussian-profile laser beam. Once the soliton is emitted, the beam is separated from the condensate dragging the emitted atoms. In (*a*) one laser beam extracts one soliton and controls its position in *z*. In (*b*) two solitons are extracted with two different beams and their paths joined. The range of times, shown on the vertical axis is from $t = 0$ to $t = 500/\nu_\perp$ in (*a*) and to $t = 1800/\nu_\perp$ in (*b*), where $\nu_\perp = 1$ kHz is the radial trapping frequency. The horizontal axis is 60 times the width *L* of the optical dipole trap that confines the condensate in the *z* direction. The figures (*c*)–(*e*) and (*f*)–(*h*) at the right display the profiles of the condensates and emitted solitons (in black continuous lines), and of the laser beams (in dotted black lines and shaded) for the different times of propagation indicated by dashed lines in (*a*) and (*b*), respectively. (The color version of this figure is included in the online version of the journal.)

laser beams (in dotted black lines and shaded) for three different times indicated with dashed lines in Figures 3(*a*) and (*b*).

The number of extracted atoms depends on the laser intensity. In Figure 4 we can see the percentage of atoms attracted and trapped by the laser tweezers as a function of the intensity of the laser for two beams of different widths w (much thinner than the longitudinal size of the BEC cloud w_c). The beam with $w \approx w_c/50$ (plotted with the symbol $\times$) is able to extract more atoms than the beam with $w \approx w_c/25$ (plotted with the symbol $*$). As can be appreciated in the plot, the number of atoms extracted decreases as the laser intensity is increased. This is due to the fact that the number of emitted solitons increases with the intensity of the laser, thus the number of atoms per soliton diminishes [25,40].

In Figure 5 we show three different numerical simulations employing beams of different intensities. Each figure corresponds with the three points labeled in Figure 4 as (a), (b) and (c). The vertical range corresponds to times from $t = 0$ to $t = 1500/\nu_\perp$.

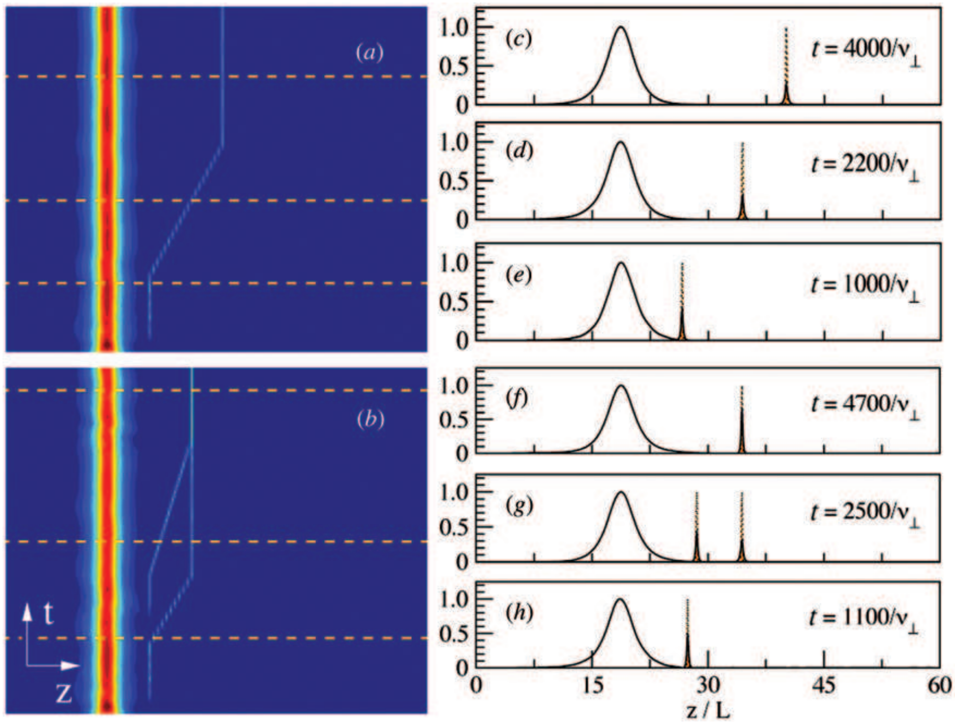

Figure 3. Same as in Figure 2 for a narrower and more intense laser beam. Vertical axis corresponds to time spanning the interval from $t = 0$ to $t = 5000/\nu_\perp$. The other parameters are as in Figure 2. The figures at the right display the profiles of the condensates and emitted solitons (in black continuous lines), and of the laser beams (in dotted black lines and shaded) at three different times of propagation indicated by the dashed lines in (*a*) and (*b*). (The color version of this figure is included in the online version of the journal.)

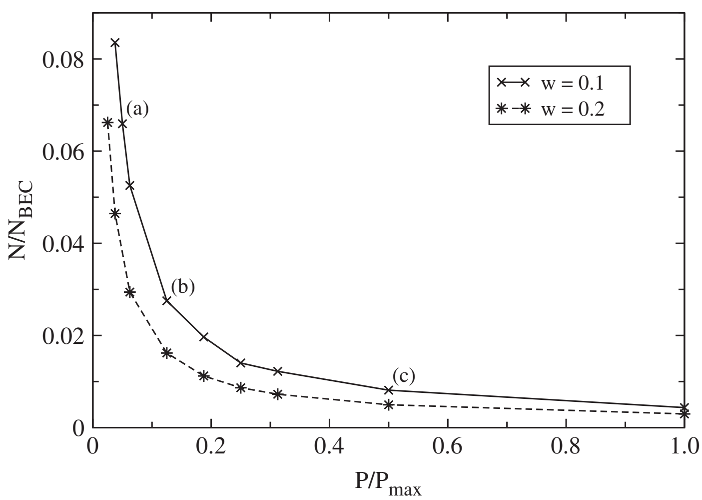

Figure 4. Dependence of the percentage of extracted atoms ($N/N_{\rm BEC}$) on the intensity of the laser beam power ($P/P_{\rm max}$) for two different widths of the outcoupling beam: $w \approx w_c/50$, plotted with the symbol $\times$ and $w \approx w_c/25$, plotted with the symbol $*$. The rest of the parameters are indicated in Figure 5.

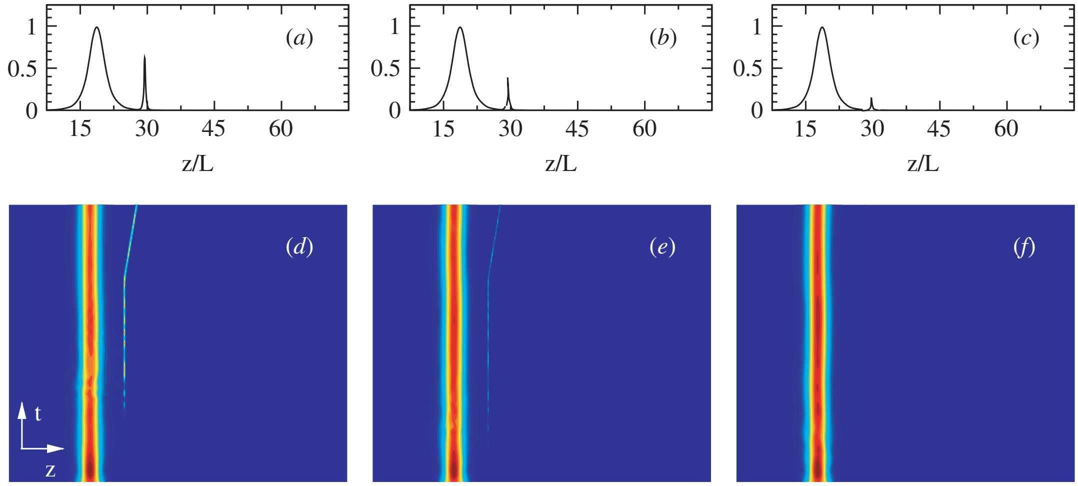

Figure 5. (*a*)–(*c*) Examples of atom extraction from a condensate corresponding to different powers corresponding to the points labeled with (a), (b) and (c) in Figure 4. Shown are the density profiles for a time propagation of $t = 1500/\nu_\perp$. The laser is separated from its original position at $t = 1100/\nu_\perp$. (*d*–*e*) Pseudocolor plot indicating the full evolution of the BEC in the time span $t \in [0, 1500/\nu_\perp]$. (The color version of this figure is included in the online version of the journal.)

For $t = 1100/\nu_\perp$ the laser is set into motion along the z axis. The upper profiles show the condensate reservoir and the extracted atom distributions.

4. Atom extraction with moving tweezers

Another interesting possibility is to use a configuration of moving lasers with variable velocities. In this case, the extraction of atoms is obtained when the laser traverses the condensate, trapping particles through its path. The number of extracted atoms N_e varies with the velocity, and with the main parameters of the beam. We have analyzed, by means of numerical simulations, the dependence of N_e on the beam intensity for several velocities in two different cases: the first one corresponding to a moving linear trapping potential and the second one corresponding to our nonlinear tweezers. Physically, the first situation consist of an optical dipole trap which does not change the value of the scattering length. We assume that in that case the laser which creates this linear trap has the same width and depth as in the nonlinear case. The only difference is that nonlinear interactions are suppressed. Our purpose with this comparison is to evaluate the effect of nonlinear interactions in the extraction procedure. To this aim, we have employed the same Schrödinger equation model as in the previous section. In all the simulations, a laser beam of width w is displaced from $z \ll 0$ to $z \gg 0$ at a fixed given velocity v, extracting a fraction of atoms N_e/N_{BEC} from the reservoir. The BEC reservoir is centered around $z = 0$.

The simulations reveal that, in the nonlinear case, the fraction of extracted atoms N_e/N_{BEC} depends crucially on the beam parameters, making the process highly controllable by changing the velocity, intensity or width of the beam. In Figure 6 we show the dependence of the percentage of extracted atoms on the velocity of the beams. The black dashed line represents the values obtained with the linear tweezers.

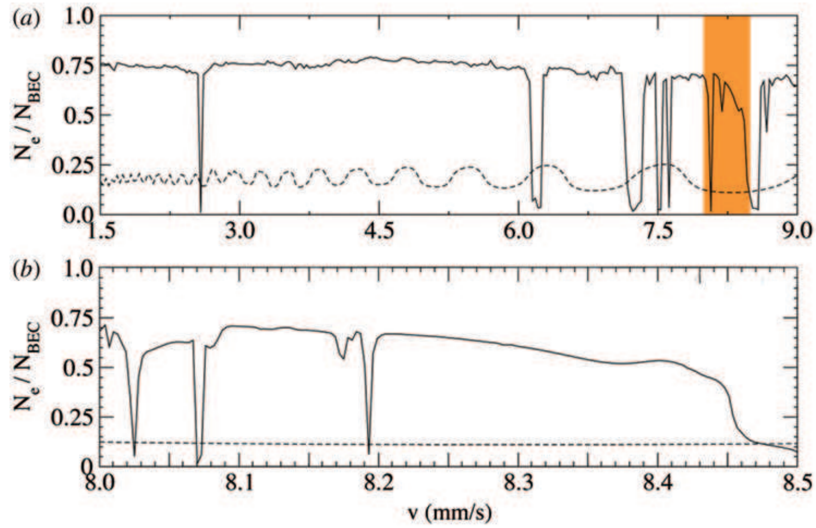

Figure 6. Top: comparison between the number of extracted atoms by nonlinear (continuous line) and linear (dashed line) tweezers for different values of the velocity. The y axis indicates the number of atoms trapped by the different optical tweezers normalized to the total number of atoms that form the initial BEC reservoir. The x axis represents the velocity of the tweezers when they traverse the condensate. Bottom: detail from the shaded region of the top plot which corresponds to velocities from $v = 8.0\,\mathrm{mm\ s^{-1}}$ to $v = 8.5\,\mathrm{mm\ s^{-1}}$. (The color version of this figure is included in the online version of the journal.)

The continuous black line shows the dependence of the extracted atoms on the velocity in the nonlinear case. As can be appreciated in the plot, the efficiency of the nonlinear optical tweezers is much higher than in the linear case. Another dramatic difference in both configurations is the presence of sharp variations of N_e at some velocities in the nonlinear configuration, allowing more control on the number of extracted atoms.

In Figure 7 we plot the variation of N_e/N_{BEC} versus w (Figure 7(a)) and the depth of the dipole optical potential V_d, measured in units of $V_0 = \hbar\nu_\perp/2$ (Figure 7(b)). In both cases, the data were obtained by fixing the velocity of the beams to $v = 5\,\mathrm{mm\ s^{-1}}$. As in Figure 6 the continuous lines refer to the nonlinear tweezers and the dashed lines to the linear ones. As can be appreciated in the captions, the behavior is similar to the variations observed in Figure 6. The effect of nonlinear interactions is the existence of sharp variations in the number of extracted atoms at certain values of the width and intensity of the laser tweezer. This adds extra control possibilities which are not accessible with linear traps.

From a practical point of view the regions with complex variations of the number of particles as a function of the parameters (for example the range of potentials around $V_d/V_0 \approx 2.0$ in Figure 7(b)) are interesting in order to provide control on very few number of atoms. This effect opens up new possibilities of achieving creation of macroscopic superposition of quantum states from arrays of Bose–Einstein condensates [42]. On the other hand, these regions are reminiscent of the fractal windows in chaotic scattering and resonances in soliton collisions typical of the interactions of nonlinear waves, which have received a lot of attention recently [46–48].

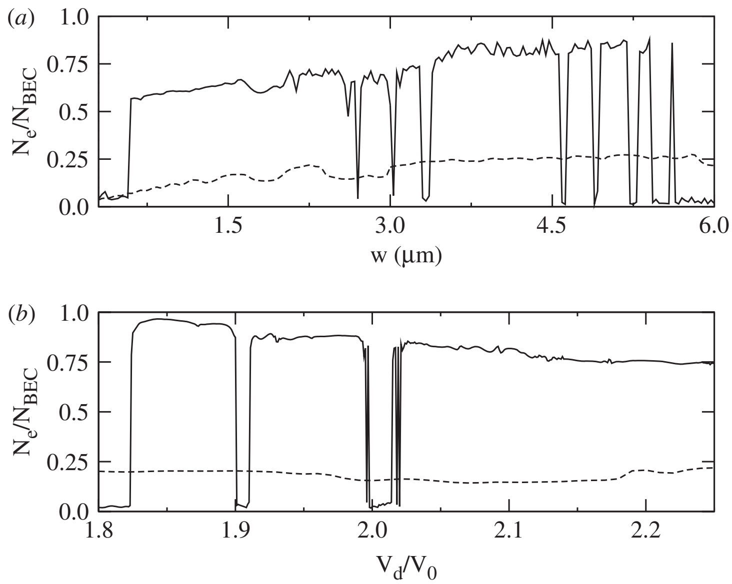

Figure 7. Fraction of atoms extracted from the reservoir as a function of (*a*) the width w and (*b*) the intensity I of the optical tweezers. The velocity was fixed to $v = 5\,\mathrm{mm\,s^{-1}}$ in all cases. The continuous lines represent the data for the nonlinear optical tweezers. The dotted lines correspond to the linear case.

5. Conclusions

In summary, we have proposed a novel mechanism for extracting atoms from a BEC reservoir. Our system uses optical tuning of nonlinear interactions between atoms to extract them from the trap.

By means of numerical simulations of the mean-field model equations, we have shown that this optically-induced spatial variation of the scattering length allows us to control the number of atoms extracted and the position of the outcoupled solitary wavepacket in a simple and robust way.

We have also compared our nonlinear tweezer concept with the action of a linear potential as the one generated by ordinary laser beams in order to illustrate the crucial effect of nonlinear interactions in the process. We have also described chaotic scattering of solitons.

Our results provide new ways to control the preparation of BECs with a controllable number of atoms and are fully testable with current BEC experiments and can be easily generalized to systems with different atomic species.

Acknowledgements

This work has been partially supported by Ministerio de Educación y Ciencia, Spain (projects FIS2004–02466, FIS2006–04190, network FIS2004–20188-E), Xunta de Galicia (project PGIDIT04TIC383001PR) and Junta de Comunidades de Castilla-La Mancha (project PAI-05-001).

References

[1] Anderson, M.H.; Ensher, J.R.; Matthews, M.R.; Wieman, C.E.; Cornell, E.A. *Science*. **1995,** *269,* 198–201.

[2] Davis, K.B.; Mewes, M.O.; Andrews, M.R.; VanDruten, N.J.; Durfee, D.S.; Kurn, D.M.; Ketterle, W. *Phys. Rev. Lett*. **1995,** *75,* 3969–3973.

[3] Roach, T.M.; Abele, H.; Boshier, M.G.; Grossman, H.L.; Zetie, K.P.; Hinds, E.A. *Phys. Rev. Lett*. **1995,** *75,* 629–632.

[4] Folman, R.; Kruger, P.; Schmiedmayer, J.; Denschlag, J.; Henkel, C. *Adv. At. Mol. Opt. Phys*. **2002,** *48,* 263–356.

[5] Leboeuf, P.; Pavloff, N. *Phys. Rev. A* **2001,** *64,* 033602-1–17.

[6] Berg-Sørensen, K.; Mølmer, K. *Phys. Rev. A* **1998,** *58,* 1480–1484.

[7] Gupta, S.; Murch, K.W.; Moore, K.L.; Purdy, T.P.; Stamper-Kurn, D.M. *Phys. Rev. Lett*. **2005,** *95,* 143201-1–4.

[8] Carpentier, A.V.; Michinel, H. *Europhys. Lett*. **2007,** *78,* 10002-1–6.

[9] Mewes, M.O.; Andrews, M.R.; Kurn, D.M.; Durfee, D.S.; Townsend, C.G.; Ketterle, W. *Phys. Rev. Lett*. **1997,** *78,* 582–585.

[10] Bloch, I.; Hänsch, T.W.; Esslinger, T. *Phys. Rev. Lett*. **1999,** *82,* 3008–3011.

[11] Hagley, E.W.; Deng, L.; Kozuma, M.; Wen, J.; Helmerson, K.; Rolston, S.L.; Phillips, W.D. *Science*. **1999,** *283,* 1706–1709.

[12] Bloch, I.; Köhl, M.; Greiner, M.; Hänsch, T.W.; Esslinger, T. *Phys. Rev. Lett*. **2001,** *87,* 030401-1–4.

[13] Martin, J.L.; McKenzie, C.R.; Thomas, N.R.; Warrington, D.M.; Wilson, A.C. *J. Phys. B* **2000,** *33,* 3919–3927.

[14] Mohring, B.; Bienert, M.; Haug, F.; Morigi, G.; Schleich, W.P.; Raizen, M.G. *Phys. Rev. A* **2005,** *71,* 053601-1–10.

[15] Guerin, W.; Riou, J.F.; Gaebler, J.P.; Josse, V.; Bouyer, P.; Aspect, A. *Phys. Rev. Lett*. **2006,** *97,* 200402-1–4.

[16] Kasevich, M.; Chu, S. *Phys. Rev. Lett*. **2001,** *67,* 181–184.

[17] Unanyan, R.G.; Pietrzyk, M.E.; Shore, B.W.; Bergmann, K. *Phys. Rev. A* **2004,** *70,* 053404-1–13.

[18] Vanier, J. *Appl. Phys. B* **2005,** *81,* 421–442.

[19] Poizat, J.P.; Grangier, P. *Phys. Rev. Lett*. **1993,** *70,* 271–274.

[20] Inouye, S.; Andrews, M.R.; Stenger, J.; Miesner, H.J.; Stamper-Kurn, D.M.; Ketterle, W. *Nature*. **1998,** *392,* 151–154.

[21] Donley, E.A.; Claussen, N.R.; Cornish, S.L.; Roberts, J.L.; Cornell, E.A.; Wieman, C.E. *Nature*. **2001,** *412,* 295–299.

[22] Pérez-García, V.M.; Michinel, H.; Herrero, H. *Phys. Rev. A* **1998,** *57,* 3837–3842.

[23] Strecker, K.E.; Partridge, G.B.; Truscott, A.G.; Hulet, R.G. *Nature*. **2002,** *417,* 150–153.

[24] Khaykovich, L.; Schreck, F.; Ferrari, G.; Bourdel, T.; Cubizolles, J.; Carr, L.D.; Castin, Y.; Salomon, C. *Science*. **2002,** *296,* 1290–1293.

[25] Theis, M.; Thalhammer, G.; Winkler, K.; Hellwig, M.; Ruff, G.; Grimm, R.; Denschlag, J.H. *Phys. Rev. Lett*. **2004,** *93,* 123001-1–4.

[26] Torres, P. *Nonlinearity*. **2006,** *19,* 2103–2113.

[27] Fibich, G.; Sivan, Y.; Weinstein, M.I. *Physica D* **2006,** *217,* 31–57.

[28] Sivan, Y.; Fibich, G.; Weinstein, M.I. *Phys. Rev. Lett*. **2006,** *97,* 193902-1–4.

[29] Belmonte-Beitia, J.; Pérez-García, V.M.; Vekslerchik, V.; Torres, P. *Phys. Rev. Lett*. **2007,** *98,* 064102-1–4.

[30] Abdullaev, F.K.; Garnier, J. *Phys. Rev. A* **2005,** *72,* 061605(R)-1–4.

[31] Garnier, J.; Abdullaev, F.K. *Phys. Rev. A* **2006,** *74,* 013604-1–6.

[32] Theocharis, G.; Schmelcher, P.; Kevrekidis, P.G.; Frantzeskakis, D.J. *Phys. Rev. A* **2005,** *72,* 033614-1–9.

[33] Sakaguchi, H.; Malomed, B. *Phys. Rev. E* **2005,** *72,* 046610-1–8.

[34] Bludov, Y.V.; Konotop, V.V. *Phys. Rev. A* **2006,** *74,* 043616-1–6.

[35] Porter, M.A.; Kevrekidis, P.G.; Malomed, B.A.; Frantzeskakis, D.J. Modulated amplitude waves in collisionally inhomogeneous Bose–Einstein condensates. **2006,** arXiv.org/labs/nlin/0607009 (accessed Jul 7, 2006).

[36] Primatarowa, M.T.; Stoychev, K.T.; Kamburova, R.S. *Phys. Rev. E* **2005,** *72,* 036608-1–5.

[37] Zezyulin, D.; Alfimov, G.L.; Konotop, V.V.; Pérez-García, V.M. *Phys. Rev. A* **2007,** *76,* 013621-1–5.

[38] Gustavson, T.L.; Chikkatur, A.P.; Leanhardt, A.E.; Gørlitz, A.; Gupta, S.; Pritchard, D.E.; Ketterle, W. *Phys. Rev. Lett.* **2001,** *88,* 020401-1–4.

[39] Boyer, V.; Godun, R.M.; Smirne, G.; Cassettari, D.; Chandrashekar, C.M.; Deb, A.B.; Laczik, Z.J.; Foot, C.J. *Phys. Rev. A* **2006,** *73,* 031402(R)-1–4.

[40] Rodas-Verde, M.I.; Michinel, H.; Pérez-García, V.M. *Phys. Rev. Lett.* **2005,** *95,* 153903-1–4.

[41] Carpentier, A.V.; Michinel, H.; Rodas-Verde, M.I.; Pérez-García, V.M. *Phys. Rev. A* **2006,** *74,* 013619-1–8.

[42] Dunningham, J.A.; Burnett, K.; Roth, R.; Phillips, W.D. *New J. Phys.* **2006,** *8,* 182-1–10.

[43] Stamper-Kurn, D.M.; Andrews, M.R.; Chikkatur, A.P.; Inouye, S.; Miesner, H.J.; Stenger, J.; Ketterle, W. *Phys. Rev. Lett.* **1998,** *80,* 2027–2030.

[44] Martinkainen, J.P. *Phys. Rev. A* **2001,** *63,* 043602-1–6.

[45] See e.g. Pérez-García V.M. Liu, X.-Y. *Appl. Math. Comput.* **2003**, *144*, 215–235.

[46] Goodman, R.H.; Haberman, R. *Phys. Rev. Lett.* **2007,** *98,* 104103-1–4.

[47] Piette, B.; Zakrzewski, W.J. *J. Phys. A* **2007,** *40,* 329–346.

[48] Zhu, Y.; Yang, J. *Phys. Rev. E* **2007,** *75,* 036605-1–20.